

# Solution of a model for the oceanic pycnocline depth: Scaling of overturning strength and meridional pressure difference

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[1] We present an analysis of the model by *Gnanadesikan* [1999] for the pycnocline depth in the ocean. An analytic solution for the overturning strength as a function of the meridional pressure difference is derived and used to discuss their mutual scaling. We show that scaling occurs only in two unphysical regimes of the model. In the absence of the Southern Ocean (SO) processes, i.e., for a northern overturning cell, the volume transport is proportional to the square root of the pressure difference. Linear scaling is seen when the overturning is restricted entirely to the SO, i.e., when no northern downwelling exists. For comparison, we present simulations with the coupled climate model CLIMBER-3 $\alpha$  which show linear scaling over a large regime of pressure differences in the North Atlantic (NA). We conclude that the pycnocline model is not able to reproduce the linear scaling between its two central variables, pressure and volume transport. **INDEX TERMS:** 4532 Oceanography: Physical: General circulation; 4203 Oceanography: General: Analytical modeling; 4267 Oceanography: General: Paleoceanography. **Citation:** Levermann, A., and A. Griesel (2004), Solution of a model for the oceanic pycnocline depth: Scaling of overturning strength and meridional pressure difference, *Geophys. Res. Lett.*, 31, L17302, doi:10.1029/2004GL020678.

## 1. Introduction

[2] The meridional overturning circulation in the Atlantic is a central challenge to our understanding of global climate dynamics. *Gnanadesikan* [1999] (hereinafter referred to as G99) presented a model for the deep meridional circulation in terms of the pycnocline depth (PD). This idealised model has been under intense investigation as a possible paradigm for the meridional overturning circulation [*Gnanadesikan and Hallberg*, 2000; *Saenko and Weaver*, 2003; *Gnanadesikan et al.*, 2002]. Furthermore it has been used to investigate the qualitative importance of different physical feedbacks on the oceanic circulation [*Klinger et al.*, 2003; *Gnanadesikan et al.*, 2003; *Kamenkovich and Sarachik*, 2004]. A qualitative feature of the deep meridional overturning circulation is the scaling relation between the volume transport and the meridional density difference in the Atlantic [*Bryan*, 1987]. Picking up Bryan's scaling arguments but assuming a constant PD in the Atlantic *Rahmstorf* [1996] proposed a linear relation which he demonstrated in the oceanic general circulation model (GCM) MOM-2. *Park* [1999] and *Scott et al.* [1999] derived the same scaling in a Stommel-type box model. GCM simulations of the ocean suggest that this linear relation carries over from the density to the pressure

difference [*Hughes and Weaver*, 1994; *Thorpe et al.*, 2001]. The linear scaling relation between pressure difference and maximum overturning strength has since been demonstrated to be a robust feature in oceanic GCM simulations A. Griesel et al. (The role of eddies for the meridional pressure gradient and the strength of the Atlantic overturning circulation, manuscript in preparation, 2004; hereafter referred to as Gr04). In section 4 we present simulations with the coupled climate model CLIMBER-3 $\alpha$  further supporting these findings.

[3] The G99 model contains four physical processes which influence the PD in the ocean. The balance of the pressure gradient in the NA and the frictional forces within the boundary currents leads to an equation for the northward volume transport

$$T_n = \frac{CD}{\beta L_y^{(n)}} \cdot \frac{\Delta p}{\rho} = \frac{Cg\Delta\rho}{\rho\beta L_y^{(n)}} \cdot D^2 \equiv \gamma_n g \Delta\rho \cdot D^2 \quad (1)$$

The pressure gradient is parameterised through the density difference in the NA  $\Delta\rho$ , the north-south distance  $L_y^{(n)}$  over which the gradient occurs and the PD  $D$ .

$$\Delta p = gD\Delta\rho \quad (2)$$

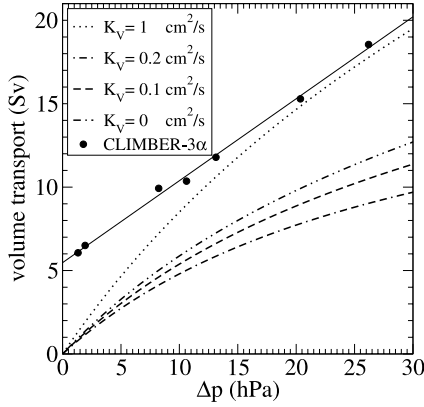
The constant  $\gamma_n$  combines  $L_y^{(n)}$  with  $\beta$ ,  $\rho$  and  $C$  (the meridional derivative of the Coriolis parameter  $f$ , the density and a proportionality constant of order one).  $g$  is the gravity constant. The quadratic dependence on  $D$  occurs due to the vertical integration in order to obtain a volume transport. In the SO the model includes the Drake passage effect through a wind-driven upwelling which does not explicitly depend on the PD  $T_s^{(e)} = (L_x \tau) / (\rho f) \equiv 2\gamma_e \cdot \tau$  and  $L_x$  are the wind stress in the SO and the circumference around Earth at the latitude of Drake Passage. Additionally G99 includes an eddy induced return flow

$$T_s^{(gm)} = L_x v_{ed} \cdot D \equiv \gamma_{gm} \cdot D \quad (3)$$

where  $v_{ed}$  is the transport velocity which G99 parameterised following *Gent and McWilliams* [1990] while we focus here on its dependence on the PD. The fourth term in the model is associated with low-latitude upwelling described by an advection-diffusion balance  $w\partial_z\rho = K_v\partial_{zz}\rho$  in the tropics which yields

$$T_u = \frac{K_v A_u}{D} \equiv \frac{\gamma_u}{D} \quad (4)$$

where  $K_v$  and  $A_u$  are the diapycnal diffusivity and the horizontal area of upwelling, respectively. All non-negative constants  $\gamma_x$  have been introduced for convenience. Note that the underlying assumption of the model is that these four process can be described using the same value  $D$  for



**Figure 1.** The analytic solution of the conceptual model for the volume transport  $T_n$  as a function of the meridional pressure difference  $\Delta p$  for different values of the diapycnal diffusivity  $K_v$ . The dots represent simulations with the coupled model CLIMBER-3 $\alpha$  which contains an oceanic GCM. The simulations exhibit a linear scaling in contrast to the conceptual model.

the PD throughout the Atlantic. Equation (1) requires furthermore that the vertical extension of the northward volume flow is also given by  $D$ . Accepting these assumptions, the conservation of volume then results in the governing equation of the model

$$0 = \gamma_n g \Delta \rho \cdot D^3 + \gamma_{gm} \cdot D^2 - 2\gamma_e \cdot D - \gamma_u \quad (5)$$

It can be shown that for all parameter settings the model has at most one solution with non-negative PD. In section 2 we give this solution analytically in terms of the volume transport  $T_n$  as a function of the pressure difference  $\Delta p$  and discuss, in section 3, its scaling with  $\Delta p$ . In section 4 we compare the results with simulations with the coupled climate model CLIMBER-3 $\alpha$ .

## 2. Solution for the Volume Transport $T_n$

[4] In order to obtain an analytic solution of the model we rewrite equation (1) to get an expression of the PD as a function of volume transport  $T_n$  and pressure difference  $\Delta p$

$$D = T_n / (\gamma_n \Delta p) \quad (6)$$

In the most interesting case of non-zero volume transport,  $T_n \neq 0$ , we can insert the equality (6) into the volume conservation equation (5) to get

$$0 = T_n + \frac{\gamma_{gm}}{\gamma_n} \frac{T_n}{\Delta p} - 2\gamma_e - \gamma_u \gamma_n \frac{\Delta p}{T_n} \quad (7)$$

Multiplying by  $T_n$  yields a quadratic equation in  $T_n$  with two solutions of which only one is non-negative

$$T_n = \frac{\gamma_n \Delta p}{\gamma_n \Delta p + \gamma_{gm}} \left( \gamma_e + \sqrt{\gamma_e^2 + \gamma_u (\gamma_n \Delta p + \gamma_{gm})} \right) \quad (8)$$

Note that despite the fact that the governing equation (5) is cubic in  $D$ , the model does have at most one physical

solution given by equation (8). The model does therefore not bear the possibility of multiple stable modes of the deep meridional overturning circulation as suggested by simulations with climate models of different complexity [Stommel, 1961; Manabe and Stouffer, 1988; Rahmstorf, 1995, 1996; Ganopolski et al., 2001; Prange et al., 2003]. This is to be expected given that the model does not include a salt-advection feedback as proposed by Stommel [1961]. Figure 1 shows the solution for different diapycnal mixing coefficients  $K_v$ . The results were obtained using the numerical values given by G99. Note that the solution (8) depends continuously on the diapycnal mixing coefficient  $K_v \propto \gamma_u$ . No change in the quality of the solution (8) occurs in the absence of the low-latitude upwelling, where

$$T_n^{(K_v=0)} = \frac{2\gamma_e \gamma_n \Delta p}{\gamma_n \Delta p + \gamma_{gm}} = \frac{\gamma_n \Delta p}{\gamma_n \Delta p + \gamma_{gm}} \cdot T_s^{(e)} \quad (9)$$

In contrast to the behaviour for vanishing  $K_v$ , the elimination of the SO processes changes the quality of the solution as can be seen from equation (8) and will be discussed in the next section.

## 3. Scaling of the Volume Transport $T_n$

[5] Next, let us discuss the scaling of the volume transport  $T_n$  with the meridional pressure difference  $\Delta p$ . First, consider the situation without the SO processes, i.e.,  $T_s^{(e)} = T_s^{(gm)} = 0$ . The scaling can be obtained from the general solution in equation (8) with  $\gamma_e = \gamma_{gm} = 0$ . More illustrative is the derivation from the original equations for the volume transport (equations (1) and (4)). The fact that the northern downwelling has to be balanced by the low-latitude upwelling  $T_n = \gamma_n D \cdot \Delta p = T_u = \gamma_u / D$  implies that

$$D^{(noSO)} = \sqrt{\frac{\gamma_u}{\gamma_n \Delta p}} \sim (\Delta p)^{-1/2}, \quad (10)$$

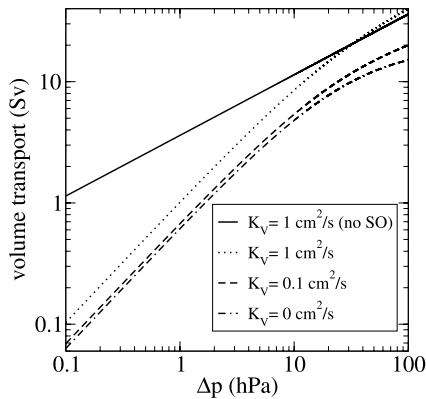
i.e., the PD decreases with increasing pressure difference in the NA. Using this expression to replace  $D$  in the parameterisation of  $T_n$  in equation (1) yields

$$T_n^{(noSO)} = \sqrt{\gamma_u \gamma_n \cdot \Delta p} \sim \sqrt{\Delta p} \quad (11)$$

In connection with equation (2) we get the scaling  $T_n^{(noSO)} \sim (\Delta p)^{1/3}$  which was derived first by Bryan [1987].

[6] Next let us add the SO winds, but neglect the eddy-induced return flow, i.e.,  $\gamma_{gm} = 0$ . The solution (8) then becomes  $T_n = \gamma_e + \sqrt{\gamma_e^2 + \gamma_u \gamma_n \Delta p}$ , which goes to a constant  $T_n \rightarrow 2\gamma_e = T_s^{(e)} = \text{const.}$  in the wind-driven limit, i.e., for small vertical diffusivity  $\gamma_u \ll \gamma_e^2 / (\gamma_n \Delta p)$ . As expected no scaling between pressure difference and volume transport is observed in this case.

[7] In order to discuss the scaling behaviour in the presence of both SO processes in the model, we plot the solutions shown in Figure 1 in double logarithmic scale in Figure 2. For small  $\Delta p$  all solutions which include the SO processes have slope one which corresponds to a linear scaling of the volume transport with the pressure difference. For comparison the solution without the SO processes from equation (11) has been included as the solid curve in Figure 2 showing the one-half slope. This result can be



**Figure 2.** The same curves as in Figure 1 in double logarithmic scale. The dashed line corresponds to the solution without the SO processes and shows a one-half scaling  $T_n \sim \sqrt{\Delta p}$ . In contrast to this the solutions which include the SO processes show a linear relationship  $T_n \sim \Delta p$  in the unphysical regime  $\Delta p \ll \Delta p_s$ .

understood from the general solution in equation (8) which also sets the scale  $\Delta p_s$  for which the linear relation holds. For  $\Delta p \ll \Delta p_s \equiv \gamma_{gm}/\gamma_n$  the solution (8) becomes

$$T_n = \frac{\gamma_n \gamma_e}{\gamma_{gm}} \left( 1 + \sqrt{1 + \frac{\gamma_u \gamma_{gm}}{\gamma_e^2}} \right) \cdot \Delta p \sim \Delta p, \quad (12)$$

i.e.,  $T_n$  is linear in the meridional pressure difference  $\Delta p$ . Using the numerical values given by G99, we obtain an estimate for the pressure scale  $\Delta p_s = 31.25$  hPa which is consistent with the scaling seen in Figure 2. Simulations with the oceanic general circulation model MOM-3 show a linear scaling of  $T_n$  with  $\Delta p$  for a variety of parameter settings, including the case of zero diapycnal mixing (A. Griesel, Gr04, 2004). The pressure scale in these simulations is of the order of  $\Delta p_s \approx 50$  hPa which is in good agreement with the above estimate.

[8] The physical meaning of the scaling regime is seen when multiplying  $\Delta p_s$  with  $D$  which gives  $T_n \ll T_s^{(gm)}$ , which means that the scaling occurs only when the circulation is completely dominated by the SO processes, i.e., when the eddy-induced return flow in the Southern Ocean is much stronger than the downwelling in the NA. This situation is not consistent with the underlying physical assumption of the model of an interhemispheric meridional overturning circulation and it does not describe the observed circulation in the ocean. From equations (6) and (12) we can see that in the linear scaling regime the pycnocline depth does not vary with the pressure difference, in contrast to the situation without SO processes (equation (10)) where  $D$  decreases with  $\Delta p$ . From equation (2) we see that for constant  $D$  the pressure difference scales in the same way as the density difference  $\Delta \rho \sim \Delta p$  making the linear scaling a simple consequence of the initial assumption that  $T_n \propto \Delta \rho$  (equation (1)).

#### 4. Comparison With Simulations

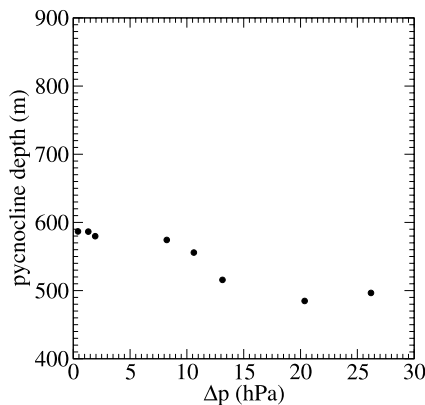
[9] The linear relationship between the maximum overturning strength and the density difference  $\Delta \rho$  which was

observed by Rahmstorf [1996] in a oceanic GCM is reflected in the parameterization of the northern downwelling in equation (1). In the conceptual model, however, it does not carry over to the pressure difference, as was shown in the previous section. In order to check this scaling we carried out simulations with the coupled climate model CLIMBER-3 $\alpha$ . The model contains an atmosphere and a sea-ice module as well as the oceanic general circulation model MOM-3. The effect of baroclinic eddies was included through a parameterization following Gent and McWilliams [1990] with a coefficient of  $\kappa_{gm} = 2.5 \cdot 10^6$  cm<sup>2</sup> s<sup>-1</sup>. For a full description of the model see M. Montoya et al. (The Earth System Model of Intermediate Complexity CLIMBER-3 $\alpha$ : Description and performance for present day conditions, manuscript in preparation, 2004). Starting from the present day equilibrium simulation with a maximum overturning strength of 12 Sv, we apply a negative salinity forcing of different strength to the NA convection sites (between 50°N and 80°N) as described by A. Levermann et al. (Dynamic sea level changes following a shutdown of the thermohaline circulation, submitted to *Climate Dynamics*, 2004). This leads to a decrease in the meridional pressure difference in the NA and therefore a weakening of the meridional overturning. A positive salinity forcing strengthens the overturning and increases the pressure difference. Figure 1 shows the simulations as black dots. The pressure was taken at a depth of 1500 m corresponding to the center of the overturning cell in the simulations. The differences were taken between the zonal average between 50°N and 80°N and the zonal average between 20°N and 30°N. This corresponds with the meridional pressure difference in the NA that enters equation (1). As seen in Figure 1 the maximum meridional overturning in the Atlantic scales linearly with the pressure difference in the NA in the simulations. The vertical diffusivity in the coupled model was kept constant at  $\kappa_v = 0.1$  cm<sup>2</sup> s<sup>-1</sup>. Thus the simulations correspond to the dashed solution curve in Figure 1. Simulations and conceptual model do neither agree quantitatively using the values suggested by G99 nor is the qualitative behaviour of the two main quantities (pressure and volume transport) reproduced in the conceptual model. These results are supported by recent findings by A. Griesel (Gr04, 2004) with an oceanic GCM. Their work shows that the linear scaling between pressure and overturning strength is a robust feature. It is independent of changes to various parameters including the Gent and McWilliams diffusivity coefficients. In order to emphasize the fact that the linear scaling  $T_n \sim \Delta p$  corresponds to constant  $D$  we plot in Figure 3 the PD as defined by G99 for our simulations. In contrast to G99 in an OGCM we find in our coupled model no significant variation of the PD for varying pressure difference.

#### 5. Conclusions

[10] By giving an analytic expression for the meridional overturning strength  $T_n$  as a function of the meridional pressure difference  $\Delta p$ , we discuss the scaling of the two main quantities of the conceptual model introduced by G99. The model exhibits two scaling regimes which both correspond to unphysical situations. Linear scaling occurs in a situation where the eddy-induced return flow is much





**Figure 3.** Pycnocline depth for the simulations shown in Figure 1 as a function of the meridional pressure difference  $\Delta p$  in the NA. Definition and displayed depth range were taken as by G99.

stronger than the northern downwelling. This corresponds to a circulation which is localized entirely in the SO and in which all downward volume transport is due to the eddy-induced return flow. This situation is inconsistent with the physical assumption of an interhemispheric overturning cell underlying the model and the isopycnal nature of the return flow.

[11] The second scaling regime corresponds to a purely northern cell where the upwelling takes place entirely in low latitudes, described by an advection-diffusion balance. In this case the overturning is proportional to the square root of the pressure as reported by Bryan [1987]. The scaling was checked using the coupled climate model CLIMBER-3 $\alpha$  with a parameter setup comparable to the conceptual model, i.e., including effects of baroclinic eddies following Gent and McWilliams [1990] and a vertical diffusivity of  $\kappa_h = 0.1 \text{ cm}^2 \text{ s}^{-1}$ . The simulations exhibit a linear scaling and therefore support previous studies [Hughes and Weaver, 1994; Rahmstorf, 1996; Thorpe et al., 2001] with comprehensive climate models. The PD does not vary significantly as a function of the pressure difference in our simulations.

[12] We conclude that the conceptual model of the PD can not reproduce the scaling between its central variables, the pressure and the volume transport. Besides possible criticism regarding the specific parameterizations of the four physical processes contained in the model, the assumption of a universal  $D$  for all these processes seems questionable.

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